

# Generating Single Peaked Votes

Toby Walsh

*NICTA and UNSW, Sydney, Australia, toby.walsh@nicta.com.au*

---

## Abstract

We discuss how to generate singled peaked votes uniformly from the Impartial Culture model.

---

## 1. Introduction

Within computational social choice, there is increasing attention away from the worst-case complexity of manipulation and control of voting rules, and focus on performance on average (for example, [7, 12, 4, 9, 10, 11]) and in practice (using, for example, resources like PrefLib [6]). There is also an increasing focus on identifying tractable special cases using tools like fixed parameter tractability (e.g. [1, 5]) and domain restrictions (e.g. [8, 3]). One of the most common domain restrictions considered in social choice theory is that of single peaked preferences. In this note, we discuss how to sample single peaked votes uniformly, correcting a common mistake found in the literature.

## 2. Generating single peaked votes

We suppose that the candidates can be ordered from left to right. An agent's preferences are single peaked when the agent prefers alternatives closer to their most preferred candidate. How do we generate single peaked votes uniformly, that is, under the Impartial Culture (IC) model? Suppose candidates are represented by integer points from 1 to  $n$  and votes are single peaked along this line. For example, for  $n = 3$ , we have 4 different single peaked votes:  $1 > 2 > 3$ ,  $2 > 1 > 3$ ,  $2 > 3 > 1$  and  $3 > 2 > 1$ . Notice that half of these single peaked votes end in 3 and, excluding the last ranked candidate 3, are themselves single peaked votes from 1 to 2. On the other hand, the other half of these single peaked votes end in 1 and, excluding the last ranked candidate 1, are single peaked votes from 2 to 3.

In general, given candidates at integer points on the line 1 to  $n$ , there are  $2^{n-1}$  different singled peaked votes. Half of all these single peaked votes end in  $n$  and are made up of all the single peaked votes from 1 to  $n - 1$  augmented with  $n$  at their end. The other half of these single peaked votes end in 1 and are made up of all the single peaked votes from 2 to  $n$  augmented with 1 at their end. This motivates the following recursive procedure which uniformly generates one of the  $2^{b-a}$  possible single peaked votes in the interval  $[a, b]$ . The vote is returned as a rank ordered list of integers.

```

GENSINGLEPEAK( $a, b$ )
1  if  $a = b$ 
2    then return []
3  else if  $\text{coin} - \text{toss} = \text{heads}$ 
4    then return Append(GENSINGLEPEAK( $a + 1, b$ ), [ $a$ ])
5    else return Append(GENSINGLEPEAK( $a, b - 1$ ), [ $b$ ])

```

Other methods have been proposed to generate singled peaked votes that sample from a different distribution. For example, Conitzer generated single peaked preferences by randomly picking a peak, and then randomly choosing the next highest alternative to the left or right of the positions currently ranked [2]. With 3 candidates, there is a  $\frac{1}{3}$  chance that Conitzer's generator will return the votes  $2 > 1 > 3$  or  $2 > 3 > 1$ , but the IC model has a  $\frac{1}{2}$  chance to generate these votes. Similarly, there is a  $\frac{1}{3}$  chance that Conitzer's generator will return the vote  $1 > 2 > 3$  (and a  $\frac{1}{3}$  chance that it will return the vote  $3 > 2 > 1$ ), but the IC model has only a  $\frac{1}{4}$  chance to generate this vote. More generally, the IC model is more likely to return votes with peaks in the middle of the left-right spectrum than Conitzer's generator. On the other hand, Conitzer's generator is more likely to return votes with peaks at the ends of the left-right spectrum than the IC model. Asymptotically, the difference between the two models is extreme. For example, the probability that the IC model returns the single peaked vote  $1 > \dots > n$  is exponentially small ( $\frac{1}{2^n}$ ) whilst Conitzer's generator returns this vote with a much greater probability ( $\frac{1}{n}$ ). Note that as  $n$  goes to infinity, the ratio between these two probabilities goes to zero.

### 3. Acknowledgements

Toby Walsh is supported by the Australian Department of Communications, the ARC, and the Asian Office of Aerospace Research and Development.

### References

- [1] N. Betzler and J. Uhlmann. Parameterized complexity of candidate control in elections and related digraph problems. *Theoretical Computer Science*, 410(52):5425–5442, 2009.
- [2] V. Conitzer. Eliciting single-peaked preferences using comparison queries. *Journal of Artificial Intelligence Research*, 35, 2009.
- [3] P. Faliszewski, E. Hemaspaandra, L.A. Hemaspaandra, and J. Rothe. The shield that never was: societies with single-peaked preferences are more open to manipulation and control. In A. Heifetz, editor, *Proceedings of the 12th Conference on Theoretical Aspects of Rationality and Knowledge (TARK-2009)*, pages 118–127, 2009.

- [4] E. Friedgut, G. Kalai, and N. Nisan. Elections can be manipulated often. In *Proc. 49th FOCS*. IEEE Computer Society Press, 2008.
- [5] L.A. Hemaspaandra, R. Lavaee, and C. Menton. Schulze and ranked-pairs voting are fixed-parameter tractable to bribe, manipulate, and control. *CoRR*, abs/1210.6963, 2014. Extended abstract appears in Proceedings of AAMAS 2013.
- [6] N. Mattei and T. Walsh. PrefLib: A library of preference data. In *Algorithmic Decision Theory, Third International Conference, (ADT 2013)*, Lecture Notes in Artificial Intelligence. Springer, 2013.
- [7] A. D. Procaccia and J. S. Rosenschein. Average-case tractability of manipulation in voting via the fraction of manipulators. In Edmund H. Durfee, Makoto Yokoo, Michael N. Huhns, and Onn Shehory, editors, *Proceedings of 6th International Joint Conference on Autonomous Agents and Multiagent Systems (AAMAS-07)*, pages 718–720. IFAAMAS, 2007.
- [8] T. Walsh. Uncertainty in preference elicitation and aggregation. In *Proceedings of the 22nd National Conference on AI*. Association for Advancement of Artificial Intelligence, 2007.
- [9] T. Walsh. Where are the really hard manipulation problems? The phase transition in manipulating the veto rule. In *Proceedings of 21st IJCAI*, pages 324–329. International Joint Conference on Artificial Intelligence, 2009.
- [10] T. Walsh. An empirical study of the manipulability of single transferable voting. In Helder Coelho, Rudi Studer, and Michael Wooldridge, editors, *Proc. of the 19th European Conference on Artificial Intelligence (ECAI-2010)*, volume 215 of *Frontiers in Artificial Intelligence and Applications*, pages 257–262. IOS Press, 2010.
- [11] T. Walsh. Where are the hard manipulation problems? *Journal of Artificial Intelligence Research*, 42:1–39, 2011.
- [12] Lirong Xia and Vincent Conitzer. Generalized scoring rules and the frequency of coalitional manipulability. In *EC '08: Proceedings of the 9th ACM conference on Electronic commerce*, pages 109–118, New York, NY, USA, 2008. ACM.